I. MATHEMATICAL APPROACH FOR COOKING A RICE GRAIN

To model the cooking process of a rice grain five relevant elements need to be taken into account:

· The rice grain is at a constant temperature T.

· Heating is diffusing faster than moisture through the rice grain[9].

· Linear diffusion is a good approximation for heating but it is not for wetting which is described by non-linear diffusion[4,5,9].

· Rice grains swell significantly during absorption which implies that diffusion with moving boundaries needs to be considered[4,5].

· If temperature is high enough granules will undergo gelatinisation and that implies a simultaneous problem of diffusion and a 1st order irreversible reaction[2,4].

· The variation of the diffusivity (D) follows an Arrehnius law with temperature[11].

Several models have been proposed taking into account only one or two of the above elements[2,8]. However, swelling and gelatinisation need to be considered in order to properly estimate wetting times. Mathematically, swelling and gelatinisation processes have been studied separately because the timescales for heating and wetting are significantly different[9].

A. Gelatinisation model

The present gelatinisation model has been built considering the following assumptions that have been made according to the literature review performed:

1_ The moisture front coincides with the gelatinisation front. The rice grain will be a sphere of equivalent radius b. The equivalent radius is the radius of the sphere that has the same volume as the grain[10].

2_ The front is sharp and delimits the interface between virgin starch and gelatinised starch. The front is located at a(t) and its position varies with time.

3_ At the front, the moisture content equals the critical moisture content needed for irreversible gelatinisation (M_{G}), which is temperature dependant, see eq. 2

4_ The flux of water has to be sufficient to gelatinise starch at the moisture front, see eq. 3

5_ Moisture follows non-linear diffusion[1] for $r > a$, see eq. 4

6_ Initially, the moisture content (M_0) is constant. Outside the rice grain the moisture content is also constant at any time (M_1), see eq. 5

The moisture content (M) in this project refers to the wet ratio (M_{w}) which is defined as the ratio of the weight of water to the total weight (dry grain weight plus water weight). It goes from 0 to 1. Whereas dry ratio (M_{d}) refers to the ratio of the mass of water to the weight of the rice grain. Both are related by:

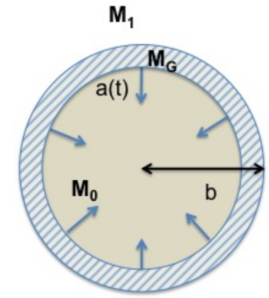


FIG. 1. Schematic representation of a spherical rice grain of radius b. The moisture front located at a(t) moves towards the center of the grain.

$$M_{\mathrm{d}} = \frac{M_{\mathrm{w}}}{1 - M_{\mathrm{w}}} \tag{1}$$

Following the indications above, the model will consist of the following equations:

$$M(a,t) = M(T_{\mathrm{G}}) \equiv M_{\mathrm{G}} \tag{2}$$

$$\frac{da}{dt} = \frac{-D\nabla M}{M_{\mathrm{G}} - M_0} \tag{3}$$

$$\frac{\partial M}{\partial t} = \nabla \cdot (D(M)\nabla M) \quad for \;\; r > a \tag{4}$$

$$M(r,0) = M_0 \qquad M(b,t) = M_1 \tag{5}$$

The expression in eq. 3 can be interpreted in terms of the mass conservation across the front discontinuity[5]. A different interpretation that leads to the same expression is that the flux of water required to move the front (left-hand term in eq. 3) equals the velocity of water arriving to the front[9]. The right-hand side can be understood from diffusion theory. It is obtained from the hypothesis that the rate transfer of matter per unit area (F) is proportional to the normal component of the concentration gradient with respect to the surface. Then for 1D, $F = -D\dfrac{\partial M}{\partial x}$ and since flux is concentration difference times velocity of the fluid we can conclude that the velocity of water inside a stationary solid is $-D\frac{(\nabla M)}{\triangle M}$.

The non-linearity in eq. 4 is due to the dependence of the moisture diffusivity (D) on M for starch-water solutions.

$$D = D_0 e^{\delta M} \tag{6}$$

, where δ and D_0 are both constants determined experimentally, we will consider $D_0 = 1.43 \cdot 10^{-7} cm^2/s$ and

$\delta = 5.22$ provided by Malcolm et al.[5] by fitting the experimental results to a single exponential.

Other expressions for D have been proposed by different authors[6,7]. Although the analytical expression for D is not clear its dependence with moisture has been experimentally proved[3] and most of the authors coincide with the expression given in eq.6. A similar one was proposed by Gomi et al.[7] after performing measurements of ground rice starch with water. The diffusivity was fitted to the expression:

$$D = -1.9 \cdot 10^{-5} + 6.58 \cdot 10^{-6} e^{M} + 8.94 \cdot 10^{-6} e^{M^2} \quad (7)$$

The condensed expression in eq.6 is similar to the experimental fit as shown in FIG.2

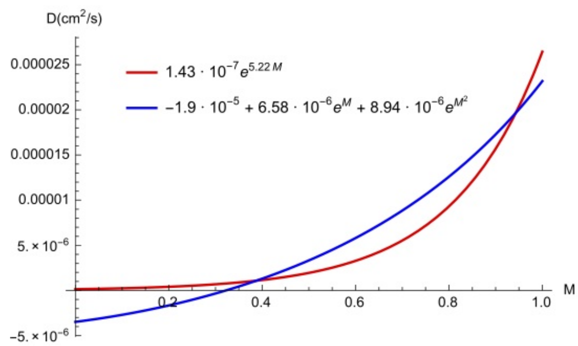


FIG. 2. The expression obtained by Gomi et al.(blue) is similar to the one we will use but returns negative diffusivities for $M \lesssim 0.4$

This set of equations has been solved for 1-dimension in Cartesian coordinates under steady-state conditions[9]. We will solve them considering that moisture is in a steady state and using spherical coordinates.

$$\frac{\partial M}{\partial t} = 0 \Rightarrow r^2 D(M) \frac{\partial M}{\partial r} = A(t) \quad (8)$$

To linearise the above partial differential equation we will use a Kirchhoff transformation

$$\Gamma = \int_{M_0}^{M} D(\omega)\, \mathrm{d}w. \quad (9)$$

We introduce (9) into the last equality of (8) by means of the Leibniz's rule for differentiation under the integral sign:

$$\frac{\partial}{\partial x}\left(\int_{a(x)}^{b(x)} f(x,y)\,\mathrm{d}y\right) = \int_{a(x)}^{b(x)} \partial_x f(x,y)\,\mathrm{d}y + f(b,x)b' - f(a,x)a' \quad (10)$$

where a' and b' are the derivatives of a and b with respect to x. Then,

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Gamma}{\partial r}\right) = 0 \Rightarrow r^2 \frac{\partial \Gamma(r)}{\partial r} = A(t) \quad (11)$$

,where A is only a function of time. If we integrate the expression above between (r, a) and ($\Gamma[M]$, $\Gamma(M_{\mathrm{G}})$) respectively:

$$\Gamma(M) - \Gamma(M_{\mathrm{G}}) = \frac{-A(t)}{r} + \frac{A(t)}{a(t)} \quad (12)$$

To find A(t) we will use the boundary condition $M(b,t) = M_1$. Thus, we will obtain:

$$A(t) = \frac{\Gamma(M_1) - \Gamma(M_{\mathrm{G}})}{\frac{1}{a(t)} - \frac{1}{b}} \quad (13)$$

Then, we can express the gelatinisation front in terms of the Kirchhoff transformation of moisture:

$$\frac{\Gamma(M) - \Gamma(M_{\mathrm{G}})}{\Gamma(M_1) - \Gamma(M_{\mathrm{G}})} = \frac{\frac{-1}{r} + \frac{1}{a(t)}}{\frac{1}{a(t)} - \frac{1}{b}} \quad (14)$$

Using the expression given in eq. 6 for diffusivity:

$$\Gamma = \frac{D_0}{\delta}[e^{\delta M} - 1] \quad (15)$$

Then, from the equation for the time evolution of the gelatinisation front (eq. 2) and the expression in eq. 8, we find that $D\nabla M = A(t)\frac{1}{r^2}$ and using the expression for A(t) we obtain the following equality:

$$\frac{\mathrm{da}}{\mathrm{dt}} = \left[\frac{-1}{r^2} \frac{1}{M_{\mathrm{G}} - M_0} \frac{\Gamma(M_1) - \Gamma(M_{\mathrm{G}})}{\frac{1}{a(t)} - \frac{1}{b}}\right]_{r=a(t)} \quad (16)$$

After rearranging the expression above and defining χ:

$$\chi \equiv \frac{(\Gamma(M_1) - \Gamma(M_{\mathrm{G}}))}{M_{\mathrm{G}} - M_0} \quad (17)$$

we perform an indefinite integral on both sides of expression 16. Thus,

$$\frac{a^2 b}{2} - \frac{a^3}{3} = -\chi b t + K_1 \quad (18)$$

,where K_1 is the integration constant that we will find by using that a(0) = b.

The moisture front location with time is a polynomial of degree three of the form:

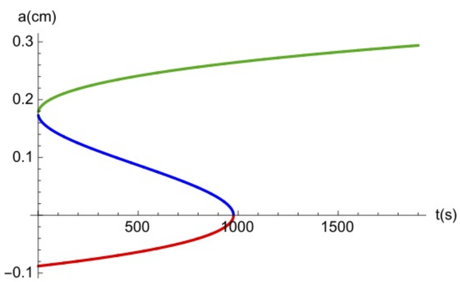
$$-2a^3 + 3ba^2 - b^3 + 6\chi b t = 0 \quad (19)$$

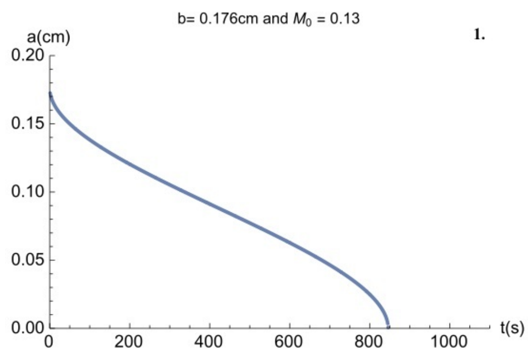
To solve the cubic equation we can use Cardano formula[12] to obtain an analytical expression for a(t):

$$a = \sqrt[3]{\frac{b^3}{8} + \frac{6\chi bt - b^3}{4} + \sqrt{\frac{18\chi^2 t^2 b^2 - 3\chi b^4 t}{8}}} + \sqrt[3]{\frac{b^3}{8} + \frac{6\chi bt - b^3}{4} - \sqrt{\frac{18\chi^2 t^2 b^2 - 3\chi b^4 t}{8}}} + b/2 \quad (20)$$

The equation has three solutions as seen in FIG 3.

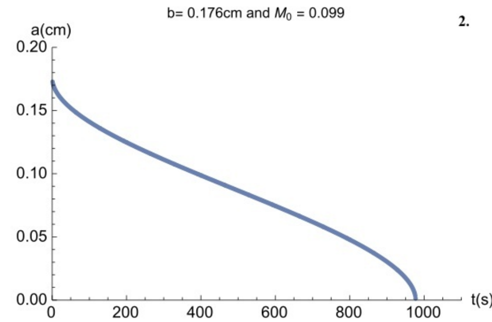
To obtain a more intuitive solution we will evaluate the above expression using Mathematica. The time needed for the front to arrive at the centre of the grain can be regarded as the cooking time. In this model, that time depends on the radius of the grain, the moisture boundary conditions and the temperature by means of the diffusivity constant, as shown in FIG 3.1 and FIG 4.



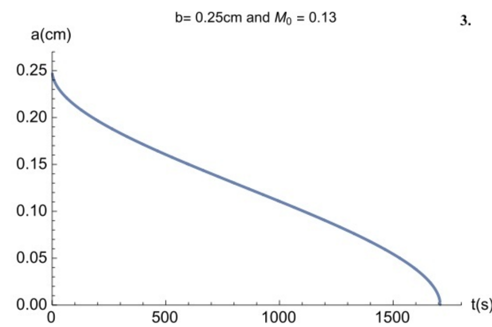


(b)Front evolution with time for b=0.176 cm and M_0 = 0.13, the cooking time is 13.75 min

FIG. 3. From three solutions of the cubic equation, the green one moves away from the grain while the red one takes only negative values.We will study the third solution (blue) that goes from a=b to a=0 in three different conditions determined by b and M_0.



(a)Front evolution with time for b=0.176 cm and $M_0 = 0.099$, the cooking time is 14.80 min



(b)Front evolution with time for b=0.25 cm and $M_0 = 0.13$, the cooking time is 28.30 min

FIG. 4. Different results for different values obtained in the literature for b and M_0

[1] Jacob Bear. *Dynamics of fluids in porous media.* Courier Corporation, 2013.

[2] Marcelo O. Bello, Marcela P. Tolaba, and Constantino Suarez. Water absorption and starch gelatinization in whole rice grain during soaking. *LWT-Food science and Technology*, 40(2):313–318, 2007.

[3] Zhongli Pan Bhagwati Prakash, Gokhan Bingol. Moisture diffusivity in rice components during absorption and desorption. *Drying Technology: An International Journal*, 2011.

[4] John Crank. *The mathematics of diffusion.* Oxford university press, 1979.

[5] Malcolm J. Davey, Kerry A. Landman, Mark J. McGuinness, Wellington, and Hong N Jin. Mathematical modeling of rice cooking and dissolution in beer production. *AIChE journal*, 48(8):1811–1826, 2002.

[6] Y. Gomi, Fukuoka M., T. Mihori, and H. Watanabe. Effect of temperature and moisture content on water diffusion coefficients in rice starch/water mixtures. *Food Science Technology*, 2(3), 1996.

[7] Y. Gomi, Fukuoka M., T. Mihori, and H. Watanabe. the rate of starch gelatinization as observed by pfg-nmr measurement of water diffusivity in rice starch/water mixtures. *Journal of Food Engineering*, 36(6):359–369, 1998.

[8] S.H. Lin. Water uptake and gelatinization of white rice. *LWT-Food Science and Technology*, 26(3):276–278, 1993.

[9] P. McGowan and M. McGuiness. Modelling the cooking process of a single cereal grain. *Proceedings of the Mathematics-in-Industry Study Group*, 1996.

[10] J. Shanthilal and C. Anandharamakrishnan. Computational and numerical modeling of rice hydration and dehydration. *Trends in Food Science & Technology*, 31(2): 100–117, 2013.

[11] Mahir Turhan and Sevim Sağol. Abrupt changes in the rates of processes occurring during hydrothermal treatment of whole starchy foods around the gelatinization temperature. *Journal of Food Engineering*, 62(4):365–371, 2004.

[12] Eric W. Weisstein. Cubic formula, 2002.